An analysis between kolmogorov scale and molecular mean free path

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As is known, Kolmogorov's analysis straightforward gives the following result,

$$\frac{\eta}{l} = \frac{\left(\nu^3/\varepsilon\right)^{1/4}}{l}, \tag{1}$$

where ν is the kinematic viscosity of the fluid, and ε is the average rate of dissipation of turbulence kinetic energy per unit mass. Qualitatively, if we assume that the dissipation rate can be characterized by local flow variables, then it gives,

$$\frac{\eta}{l} \sim Re_l^{-3/4}, \tag{2}$$

where l is the characteristic scale of eddy.

Also, the gas kinetic theory describes the Knudsen relationship with respect to Mach and Reynolds numbers,

$$Kn = \frac{Ma}{Re}\sqrt{\frac{\gamma\pi}{2}}. \tag{3}$$

As a result, the ratio of the Kolmogorov scale over molecular mean free path can be derived as,

$$\frac{\eta}{\lambda} \sim \frac{Re_l^{1/4}}{Ma}. \tag{4}$$

Assume a general turbulence condition with $Re_l = 100000$ and $Ma = 0.5$, it gives,

$$\frac{\eta}{\lambda} \simeq 70. \tag{5}$$